Perfect powers in arithmetic progression

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The problem of perfect powers forming an arithmetic progression has a very long history, going back (at least) to Fermat and Euler. More generally, one can take the product of terms of an arithmetic progression, and ask when it yields an "almost" perfect power. That is, we consider the diophantine equation

$$n(n+d)...(n+(k-1)d) = by^l \tag{1}$$

in positive integers n, d, k, b, y, l with $k, l \geq 2$, $\gcd(n, d) = 1$ and $P(b) \leq k$. Here $P(u)$ stands for the greatest prime factor of u if $u > 1$ is an integer, and $P(1) = 1$.

The literature of equation (1) is extremely rich. The equation and its various generalizations have been studied from many aspects, by several mathematicians. In the talk we give an overview of the topic and indicate the most important research directions and results of the area. We shall give a special emphasize of recent results which yield complete solution of equation (1) in certain cases. We shall briefly outline the methods and techniques used in the proofs of these results, such as the theory of modular forms, the Chabauty method, and local and combinatorial arguments.